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Heat Transfer for Highly Cooled Supersonic Turbulent Boundary Layers

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Nomenclature

C_f = local skin-friction coefficient, $\tau_w/(\rho_e u_e^2/2)$
C_h = local Stanton number, $\dot{q}/\rho_e u_e (h_{aw}-h_w)$
h = static enthalpy
M = Mach number
M_s = Mach number of incident shock wave in shock tube
Re_v = Reynolds number, $\rho_e u_e x_v/\mu_e$
T = absolute temperature
ρu = density-velocity product
x_v = distance from virtual origin of turbulence
μ = coefficient of viscosity

Subscripts

aw = adiabatic wall
e = boundary-layer edge
m = based on measured values
p = predicted quantity
w = wall

Introduction

NUMEROUS efforts have been made to develop simple prediction schemes for skin friction and to evaluate Reynolds analogy factors $2C_h/C_f$ for turbulent supersonic and hypersonic boundary-layer flows on flat plates or equivalent zero pressure gradient surfaces. Such correlation schemes, which are useful for predicting turbulent heat transfer, are available for somewhat limited combinations of Mach number and wall cooling.[1] In this Note, a correlation of heat-transfer data is established for zero-pressure-gradient turbulent boundary layers with high wall cooling ($T_w/T_{aw}<$ 0.3) occurring in flows with low supersonic Mach numbers. This flow regime has not been investigated in wind tunnels and related facilities because conditions of high wall cooling at low supersonic Mach numbers cannot be produced without elaborate means of model cooling.

However, such flows can be generated in shock tubes. In shock tubes, the test Mach number M_e and the wall cooling variable h_w/h_{aw} (used in place of T_w/T_{aw} to include real gas effects) are functions of the primary shock Mach number M_s. The flows are characterized by high wall cooling and low supersonic Mach numbers. For example, in air, for the M_s range $3\leq M_s\leq 16$, $1.4\leq M_e\leq 3$ and $0.30\geq (h_w/h_{aw})\geq 0.01$. Although M_e and h_w/h_{aw} cannot be varied independently using room temperature models, h_w/h_{aw} can be varied over a wide range, whereas the corresponding range of M_e is relatively narrow.

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Heat-transfer measurement techniques for shock-tube flows are well known; however, techniques for skin-friction measurement in shock-tube flows are not presently available. Therefore, Reynolds analogy factors cannot be established by simultaneous measurements of heat transfer and skin friction. In view of the desirability of retaining the Reynolds analogy approach, the correlation presented here is based on predicted skin-friction values and measured heat-transfer rates.

Correlation Method

Reynolds number based on boundary-layer edge conditions and the distance from the virtual origin of turbulence is employed in the prediction of the skin friction coefficient C_f. The procedure is to transform this Reynolds number Re_v to an incompressible form $\overline{Re_v}$ by the transformation relation $\overline{Re_v}=F_x Re_v$. The corresponding $\bar{C}_f$ is next determined from an accepted $\bar{C}_f$ vs $\overline{Re_v}$ relation.[2] The compressible C_f corresponding to Re_v is then determined from the relation $\bar{C}_f=F_c C_f$. The functions F_x and F_c are unique to the prediction method. Three well-known skin-friction prediction formulations are used here: those of Sommer and Short,[3] Spalding and Chi,[4] and the Van Driest (II) method.[5] The functions F_x and F_c are presented in Ref. 1 for each of these procedures for flows in which real gas effects are not generally important.

To include the real gas effects inherent in shock-tube flows, all of the temperature ratios in the transformation functions were replaced by enthalpy ratios and $u_e^2/2h_e$ was substituted for the term $(\gamma-1)\,M_e^2/2$. The recovery factor was taken as 0.9 throughout. For specified flow conditions, the outlined procedure produces three predictions for C_f that do not necessarily agree. Assumption of a Reynolds analogy factor in conjunction with any of the three skin friction prediction methods would permit prediction of Stanton numbers for comparison with corresponding experimental Stanton numbers. An alternate method would be to form a Reynolds analogy factor based on the experimental Stanton number C_{hm} and the predicted skin-friction coefficient C_{fp} and then seek a correlation in terms of this Reynolds analogy factor. The latter approach is taken here. In general the Reynolds analogy factor would be expected to depend on both M_e and h_w/h_{aw}. As previously noted, M_e varies over a rather narrow range when h_w/h_{aw} is varied over a large range. Therefore, a correlation is sought between $2C_{hm}/C_{fp}$ and the wall cooling variable h_w/h_{aw}.

Previous pertinent shock-tube studies, which present heat-transfer measurements in air for the highly cooled turbulent boundary layer on a flat plate or equivalent surface, are those of Hopkins and Nerem[6] and Jones.[7] Additional experimental data were obtained in the present study and are reported in detail in Ref. 8. Table 1 shows the range of each of the studies. Primary data acquired in shock-tube testing include incident shock speed, initial pressure and temperature of the test gas, the heat flux $\dot{q}$ measured at known x positions, and the location of the virtual origin of turbulence. For the studies described in Refs. 6 and 8, x_v was taken as the location of the boundary-layer trips used to promote turbulence, and for the Jones study,[7] x_v was taken as the location of departure from laminar flow.[8] The flow variables M_e, unit Reynolds number, and h_w/h_{aw} in the test region behind the incident shock wave and can be computed with the aid of real air charts. In turn, the corresponding values of C_{fp} and C_{hm} can be computed to produce the Reynolds analogy factor $2C_{hm}/C_{fp}$.

Results and Discussion

The results obtained from analysis of the data of the three experimental studies listed in Table 1 using the Van Driest II method to predict C_f are presented in Fig. 1. A scale of edge Mach number is also shown. Although scatter exists in the results, particularly at low values of h_w/h_{aw}, the data for the three studies generally complement each other. Three data points based on Jones's study exhibit good agreement with the

Table 1 Shock tube heat transfer studies; zero pressure gradient turbulent boundary layers

Study	Ref.	h_w/h_{aw}	M_e	M_s	Re_v
Present	8	0.24-0.13	1.5-1.8	3.24- 4.65	1.4×10^5-3.4×10^6
Jones	7	0.13-0.04	1.8-2.4	4.61- 8.10	3.2×10^5-1.8×10^7
Hopkins and Nerem	6	0.03-0.01	2.6-3.1	9.50-16.40	1.3×10^5-1.2×10^6

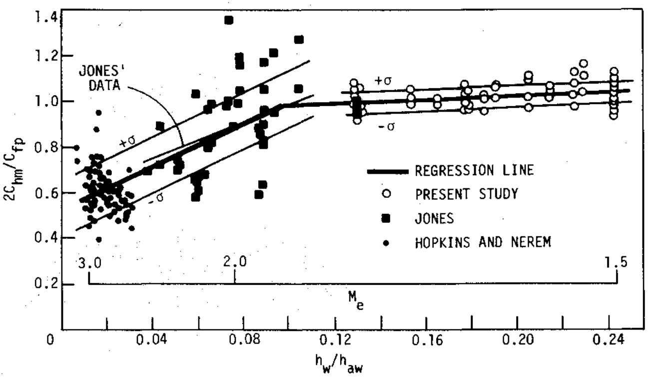

Fig. 1 Heat transfer correlation $2C_{hm}/C_{fp}$ vs h_w/h_{aw}. C_f predicted by Van Driest II method, Ref. 5.

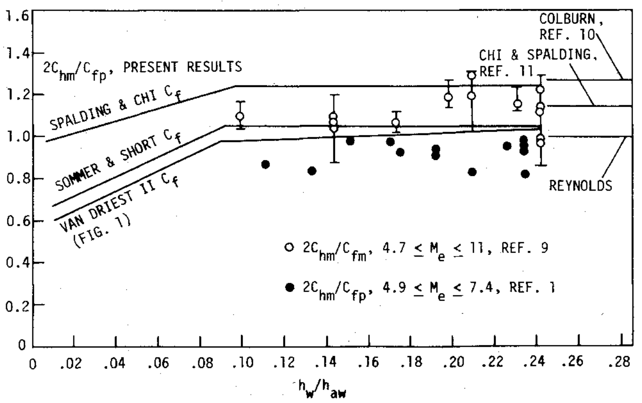

Fig. 2 Comparison of $2C_h/C_f$ values.

present results at $h_w/h_{aw}=0.13$. The data points based on Jones' study generally show a trend to the lower values of $2C_{hm}/C_{fp}$ from the study of Hopkins and Nerem. In view of the overall trend of the results, the data were divided at $h_w/h_{aw}=0.12$ for the statistical analysis. Straight lines were fit to each group of data by the method of least squares. The lines for the data for $h_w/h_{aw}>0.12$ have been extended to the line fit through the data below $h_w/h_{aw}=0.12$. The line fit through only Jones' data supports this approach. Bands indicating a departure of one standard diviation σ from the fitted lines are shown. The results in Fig. 1 are typical of those obtained by the other two methods for predicting C_f. However, different fitted curves result. Correlation coefficients from the statistical analysis of results indicate a statistical preference for the results in Fig. 1.

It is useful to compare values of $2C_{hm}/C_{fp}$ with values of Reynolds analogy factor and similar results from other studies. Such a comparison is presented in Fig. 2, which shows the fitted curves for the present results corresponding to each C_f prediction method employed. Also shown are three values for Reynolds analogy factor commonly applied in other flow regimes. Figure 2 includes Reynolds analogy factors from two other groups of experimental data obtained for hypersonic zero-pressure-gradient flows with high wall cooling. The results of Cary[9] were determined from several studies in which C_f and C_h were directly measured. The solid symbols are from Hopkins and Inouye,[1] for which C_f was predicted by the Van Driest II method and C_h was based on measured heat-transfer rates. For $h_w/h_{aw}>0.10$, the present correlations resulting from C_f predictions by the Sommer and Short and Van Driest II methods fall approximately in the middle of the hypersonic results, whereas results based on Spalding and Chi C_f values lie above and in the upper range of the hypersonic results and tend to agree with the Colburn value[10] for Reynolds analogy factor. The remaining two curves agree well with Reynolds' original value of unity for the Reynolds analogy factor. In the interest of maintaining reasonable consistency with other data for $h_w/h_{aw}>0.10$, Fig. 2 suggests that the present correlation based on the Spalding and Chi skin friction be excluded. The correlation based on the Van Driest II skin friction prediction (Fig. 1) is selected as the preferred one of the remaining two. This choice is based on the statistical preference for the fitted lines in Fig. 1 and on the observation that, for $h_w/h_{aw}>0.10$, $2C_{hm}/C_{fp}$ for this correlation is essentially unity (in agreement with the Reynolds analogy). A further convenience of this choice, although not necessarily a justification, is that the Van Driest II skin-friction prediction method with a Reynolds analogy factor of unity has been tentatively selected by Hopkins and Inouye[1] for heat-transfer prediction for hypersonic flows with moderate wall cooling ($T_w/T_{aw}>0.3$).

Due to variations in experimental results of the three studies, the correlation in Fig. 1 permits heat transfer to be predicted with greater certainty above $h_w/h_{aw}=0.12$ than below this value. Although the recommended correlation should serve to predict heat transfer for the range of variables considered, the quantity $2C_{hm}/C_{fp}$ should not be viewed as a confirmed Reynolds analogy factor since skin-friction prediction techniques have not been verified for shock-tube flows.

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Lyapunov-Designed MRAS-Linearized Error Equation Results

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Introduction

THE large volume of work in the field of model-reference adaptive control (MRAS) during the last decade has resulted in a large number of adaptive controller/observer techniques.[1,2] For those methods where asymptotic stability has been proved, it has been due to the application of Lyapunov's Second Method because of the inherent nonlinear and complex nature of these equations. The nonlinearities cause great difficulty in analyzing such adaptive systems. A concept that has met with some success in the analyzing MRAS systems is that of the linearized error characteristic equation (LECE) approach.[3-5] The purpose of this Note is to present some conjectured propositions based on results of applying the LECE concept to some published adaptive techniques. These potential theorems, in addition to being of interest in their own right, can provide insight into design and analysis of MRAS controller/observer systems. Additionally, they provide a means of demonstrating the manner in which results from Lyapunov Theory agree with classical linear stability concepts when comparing with the linearized root locus expressions using the LECE approach.

Linearized Error Equation Approach

The LECE technique is a general approach for obtaining root locus expressions for systems involving adaptation, whether an observer or controller. It relates adaptation design parameters to the adaptive error transient response. The approach consists of deriving a set of linearized error equations for each adaptive method about an operating point and manipulating these truncated linear equations into the form $1+KG(s)$ for plotting root loci. For the case of an adaptive control system, the plant is adapted to track a reference model, and for the observer the plant is fixed and an adjustable model is adapted.

The LECE concept, based on error equation analysis, has a strong theoretical basis in the case of the adaptive control, but requires clarification in the observer (or identification) case. This is because the Lyapunov design theory for observers generally requires a "frequency richness" condition on the input to insure parameter identification. In developing the LECE for observers, it is assumed constant inputs are applied. This effect can be essentially met as long as any a.c. variation on u, and their derivatives, are "small" with respect to any bias terms on u. Without the assumption of constant u, meaningful LECE results do not appear possible.

Applying the LECE technique to five published adaptation methods, one obtains a series of LECE expressions which can be used for design and analysis purposes. The details are given in Table 1.[4] Methods 1, 2, 4, and 5 are adaptive control laws, method 1 being the Winsor and Roy,[6] method 2 the Ten Cate,[7] method 4 the Gilbart et al.,[3] and method 5 the Sutherlin and Boland method.[8] Method 3 is the adaptive observer of Kudva and Narendra.[9,10]

Common to methods 1, 2, 4, and 5 is the Z factor (Table 1), which includes weighting constants q_{ij}, elements of a positive definite (p.d.) symmetric matrix Q satisfying

$$A_m^T Q + Q A_m = -C \tag{1}$$

where A_m is a stable model matrix that is to be tracked and C is a p.d. symmetric matrix. Method 3 employs a weighting constant α, $0<\alpha<2$, P is a p.d. symmetric matrix, and λ_{max} is the largest eigenvalue of P.

LECE Conjectures

Using the LECE approach on the previous methods, the resulting root locus expressions are given in Table 1. From these expressions, and some additional material, five conjectured propositions can be formulated.

Conjecture 1. If A_m is a constant, stable $n \times n$ matrix in the phase variable form, C is a p.d. symmetric $n \times n$ matrix, Q is a p.d. symmetric matrix satisfying Eq. (1), and q_{ij} is the ijth element of Q, then the roots x_i, $i=1,2,\ldots, n$ of

$$\sum_{i=1}^{n} q_{ni} x^{i-1} = 0 \tag{2}$$

satisfy $R_e\{x_i\}<0$, $i=1,2,\ldots,n$.

This particular conjecture comes about from an examination of the LECE's of methods 1, 4, and 5 of Table 1. The roots x_i represent the location of open-loop zeros of a root locus expression. Since the root locus gain can become infinite, if the LECE approach is to yield realistic results, the zeros would have to be in the open left-hand s plane for the adaptive system to be stable for large values of inputs.

From a study of the LECE of method 2 in Table 1, the following proposition can be formulated.

Conjecture 2. If A_m is a constant, stable $n \times n$ matrix in the phase-variable form for the single-input system $\dot{x}=A_m x+bu$, where $b^T=(00\ldots01)$, C is a p.d. symmetric $n \times n$ matrix, γ is a nonnegative scalar, Q is a p.d. symmetric $n \times n$ matrix satisfying Eq. (1) and q_{ij} is the ijth element of Q, then the roots x_i, $i=1,2,\ldots, n$ of

$$\sum_{i=1}^{n} [q_{ni} x^{i-1}] + \gamma\, |xI - A_m| = 0 \tag{3}$$

are such that $Re\{x_i\}<0$.

Adaptive observers designed by Lyapunov Theory also exhibit interesting stability properties, as given in the following proposition.

Conjecture 3. If α is a scalar such that $0<\alpha<2$, P is a p.d. symmetric $n \times n$ matrix, and λ_{max} is the largest eigenvalue of P, then the roots z_i, $i=1,2,\ldots,(n-1)$ of

$$(z-1) z^{n-2} + \frac{\alpha}{\lambda_{max}} \sum_{j=2}^{n} p_{jn} z^{n-j} = 0 \tag{4}$$

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